# Substructuring approach and blocked forces method: application for structure-borne vibration prediction in heavy weight assemblies

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## Abstract
Prediction of vibration levels in heavy structures enables engineers to adopt vibration mitigation measures in the initial design stages, therefore reducing costs and time taken for construction or installation. Different strategies for achieving accurate predictions range from numerical methods, such as FEA and SEA, to methods incorporating measured data. Here, we present an experimental method that combines the sub-structuring approach with the in-situ blocked forces method to characterise the key elements of an assembly. In order to implement this methodology, the assembly is subdivided into the components vibration source, isolator and receiver. These three elements are characterised by performing mobility measurements on the individual substructures and the source is additionally characterised by the blocked force measured in-situ. Predictions made using the independent component characterisations are compared with measures obtained from a whole assembly composed of a heavy structure supported on resilient isolators.

## 1 Introduction

Vibration prediction is important in buildings designed for living, working, listening and for certain types of sensitive equipment. Unlike airborne, structure borne noise prediction is very challenging due to the complexity for characterising the elements involved. Besides an important increase on research studies for predicting structure borne noise in industries such as automobile or aircraft in recent years, that trend is not so noticeable within the building industry.

Different strategies used for prediction of ground/structure borne noise range from numerical methods, which discretise the problem domain into well-defined inputs expressed as equations or parametric studies (e.g. Finite Element Analysis, Statistical Energy Analysis); to measurement based methods, that reduce the problem to transfer functions between assembly elements [1].

Due to limitations of numerical methods such as the high computational requirements and their disagreement with field data [2], there has been an increase in the development of solutions on measurement based approaches such as dynamic sub-structuring for light structures [3].

The combination of *in situ* blocked forces and sub-structuring empirical methods (described in the following section) has been proven accurate for the prediction of structure borne noise from lightweight assemblies e.g. electric pumps [4] or engine parts [5], and heavier assemblies such as micro wind turbines [6]. In order to implement this methodology, the assembly is subdivided into the components: vibration source, isolator and receiver. These three elements are then characterised by performing mobility or impedance measurements on the individual sub-structures and the source is additionally characterised by the blocked force measured in-situ [7].

Besides the good agreement shown between predictions and *in situ* measurements with lighter structures, the accuracy of the aforementioned methods for heavier structures, more representative of realistic factory machinery or building situations, is yet to be assessed and will be the focus of this work.

# 2 Theory and Methods

In order to simplify the notation used throughout the present work and to also allow its application to other situations, the assembly is subdivided into three general elements namely source (S), isolator (I) and receiver (R). The source is defined as the object or structure that generates a vibration which is propagated to a final structure known as receiver. These two elements are connected via another structure defined as the isolator, which typically provides a resilient link in order to reduce structure-borne vibration.

Furthermore, Figure 1 describes the three aforementioned elements, together with four defined interfaces that comprise the different excitation/response positions: Internal mechanisms of the source which are not part of the interface with isolator (a), the source-isolator interface (b), the isolator-receiver interface (c), and response points on the receiver away from the interface (d).

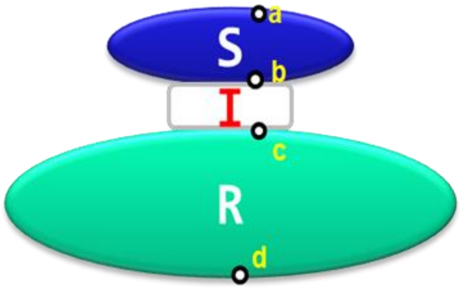


Figure 1: Source-Isolator-Receiver assembly diagram showing the different elements (S, I, R) and interfaces (a-d)

In the following sections a similar nomenclature is followed, consisting of the use of subscripts S, I, R and C for referring to the independent Source, Receiver, Isolator and the Coupled assembly respectively. Following this, two subscripts are used to denote the sets of degrees of freedom where response and excitation measurements are made respectively. As an example, $Y_{Ccb}$ refers to the transfer mobility of the coupled assembly where the response is observed below the isolator (c) when exciting at (b).

## 2.1 Blocked Force Method for active source characterisation

Blocked forces of structure borne sound sources can be obtained *in situ* as demonstrated in [7], [8], and can be defined as:

$$\bar{\boldsymbol{f}}_{Sb} = \boldsymbol{Y}_{S,bb}{}^{-1}\tilde{\boldsymbol{v}}_{Sb} \tag{1}$$

where $f_{Sb}$ is the blocked force of the source at interface b, $Y_S$ is the source mobility and $v_{sb}$ is the velocity of the free source. Over score and tilde denote blocked and free conditions respectively.

An alternative of the blocked force can also be obtained from coupled assembly measurements *in situ*, therefore not requiring free-free conditions:

$$\bar{\boldsymbol{f}}_{Sb} = \boldsymbol{Y}_{C,bb}{}^{-1}\boldsymbol{v}_{Cb} \tag{2}$$

where $f_{Sb}$ is the blocked force of the source at interface b, $Y_{C,bb}$ is the coupled source mobility and $v_{Cb}$ is the operational velocity of the coupled source at b.

Due to the fact of being an inherent characteristic of the source, the blocked force can also be used to predict the structure borne vibration when being part of different assemblies.

## 2.2 Dynamic sub-structuring

As detailed in [7], the dynamic substructuring method (DS) allows for the prediction of a coupled assembly behaviour as individual contributions of the different subsystems that are part of it. This methodology provides important advantages when faced with heavy weight structure limitations:

- It allows freedom for the characterisation of the different assembly elements, enabling the combination of approaches such as numerical methods (e.g. FEA or SEA); or empirical methods that incorporate directly measured data, which is the option presented in this work.

- A reverse application of dynamic sub-structuring, structure decoupling, allows the extraction of the properties of one of the sub-structures from data collected from the coupled assembly together with the remaining sub-structures. A useful application of this would be to avoid the need to mount an industrial press under free conditions obtaining its properties by subtracting the isolator and floor impedance from the coupled assembly.

However, the use of dynamic sub-structuring has implicit limitations that condition the reliability of the resulting predictions. To a large extent the uncertainties come from the fact that the application of matrix inversion is highly sensitive to errors, being able to affect the entire matrix when only one element is wrong before inversion [9].

Another important factor affecting the accuracy of predictions of sub-structuring is the number of degrees of freedom (DOF) accounted for. Due to the inherent complexity of interface dynamics, a lack of information regarding translational and rotational DOF can strongly influence the error between prediction and on board validation measurements, therefore justifying the need of accounting for all DOF. However, the use of resilient isolators has proven to reduce this effect in lower frequencies, being able to provide reasonably accurate predictions when neglecting rotational and in-plane forces [10].

In order to obtain the passive properties of the different sub-structures and predict their coupled behaviour, source, isolator and receiver were independently characterised (detailed in following section). The classical impedance procedure was applied [11] to the resulting accelerance matrices by constructing a block diagonal matrix which contains the independent assembly element impedance matrices:

$$\boldsymbol{Z_{BlDiag}} = \begin{bmatrix} \boldsymbol{Y_S^{-1}} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{Z_i} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{Z_i} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{Z_i} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{Y_R^{-1}} \end{bmatrix} \quad \textbf{(3)}$$

where $Y_S$ is the source mobility (3 by 3 matrix), $Z_i$ is the isolator impedance (2 by 2 matrix) and $Y_R$ is the receiver mobility including the interface d FRFs (4 by 4 matrix).

The resulting diagonal matrix (13 by 13) is pre and post multiplied by two Boolean coupling matrices in order to assure conditions of equilibrium and compatibility:

The pre-multiplicative matrix $L_f$ enforces equilibrium at the coupling DOFs, and the post-multiplicative matrix $L_v$ enforces compatibility of the coupling DOFs, whilst also accounting for the remote reference DOFs. This way, the overall coupled mobility matrix result of the application of the dynamic sub-structuring method would be given by:

$$\boldsymbol{Y_C} = \left(\boldsymbol{L_f Z_{BlDiag} L_v}\right)^{-1} \tag{4}$$

, where:

$$L_f = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

and:

$$L_v = {L_f}^T$$

# 3 Methods

In order to be able to implement a methodology that combines the use of blocked forces for active source characterisation together with dynamic sub-structuring of the entire assembly, source and receiver were characterised under free-free conditions whereas the isolator was characterised both *in situ* and under dynamic compression (MTS) using a dynamic hydraulic testing machine. Despite the use of free mounting for the source and receiver characterisation, other mountings could have been used following Eq. (2). Furthermore, the source-isolator-receiver coupled assembly was characterised and the resulting data used for validation of the different methods described in this section.

The different elements were characterised by obtaining the point and transfer accelerance frequency response functions (FRFs) derived from force excitations and acceleration responses of the individual substructures *in-situ*. The instrumentation used for these experiments consisted of 4507-B004 (B&K) single axis accelerometers as response sensors, and an 8206-001 instrumentation hammer (B&K) for performing the different structure excitations. The force and acceleration measures together with their Frequency Response Functions (FRFs) were synchronously collected using a SIRIUS acquisition card (DEWESOFT) at a sampling rate of 20000Hz with a frequency resolution of 0.6 Hz/data point.

## 3.1 Source and Receiver characterisation

The source used was a servo motor bolted to an aluminium plate, which at the same time is bolted to three steel feet used for the connection with the isolator mounts (Figure 2-top). Free-free conditions were implemented by suspending the source using elastic bungees.

Six single axis accelerometers were used for the characterisation of the source, two per foot and separated by 2.5 cm. Force excitations were applied on top of the three feet and at interface a.

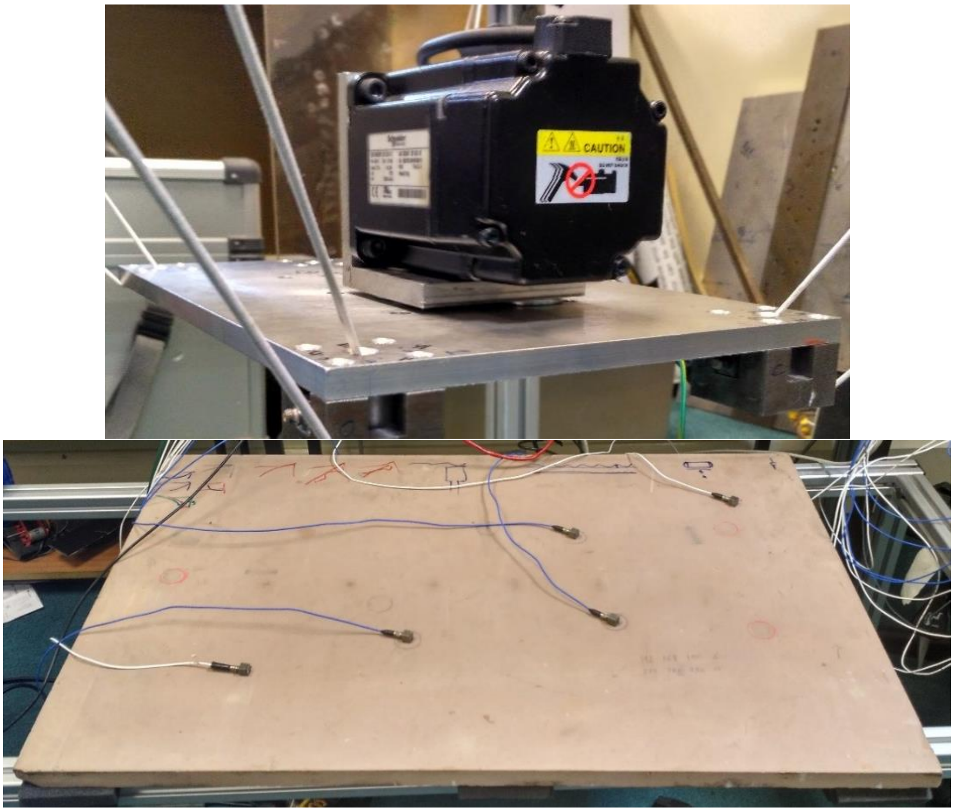


Figure 2: Free-free experiment view of Source (top) and Receiver (bottom).

A 900x600x20 mm construction slab (Figure 2-bottom) was used as the receiver structure for the presented work. Three accelerometers acquired the structure response below the three coupling points between the receiver and the isolators (Figure 4), together with two additional accelerometers positioned away from the connections (interface d). Three force excitations were applied on top of the three coupling points.

In order to proof that the blocked force method can predict the effect of the internal mechanism of a vibrating source (interface a) when coupled to a different assembly, the blocked force was calculated from the free-free independent source experiment by applying Eq. (1). The acceleration at response points d was then predicted using this blocked force together with the *in situ* accelerance obtained in the coupled experiment between interfaces b and d:

$$\boldsymbol{a}_{\boldsymbol{C},\boldsymbol{d}} = \boldsymbol{A}_{\boldsymbol{C},\boldsymbol{db}}\bar{\boldsymbol{f}}_{\boldsymbol{Sb}} \quad \textbf{(5)}$$

The acceleration predicted at d using the blocked force method Eq. (1) (5) was compared with the *in situ* obtained from the coupled experiment, and can be observed in Figure 3 in terms of narrow band frequencies (top) and in third octave bands (middle).

In order to provide a numerical expression of the accuracy of the prediction, the error was calculated in the frequency domain by using Eq. 6, and is shown graphically in Figure 3-bottom.

$$error\ (dB) = 20\log\left({}^{predicted\_response}/{}_{measured\_response}\right) \quad \textbf{(6)}$$

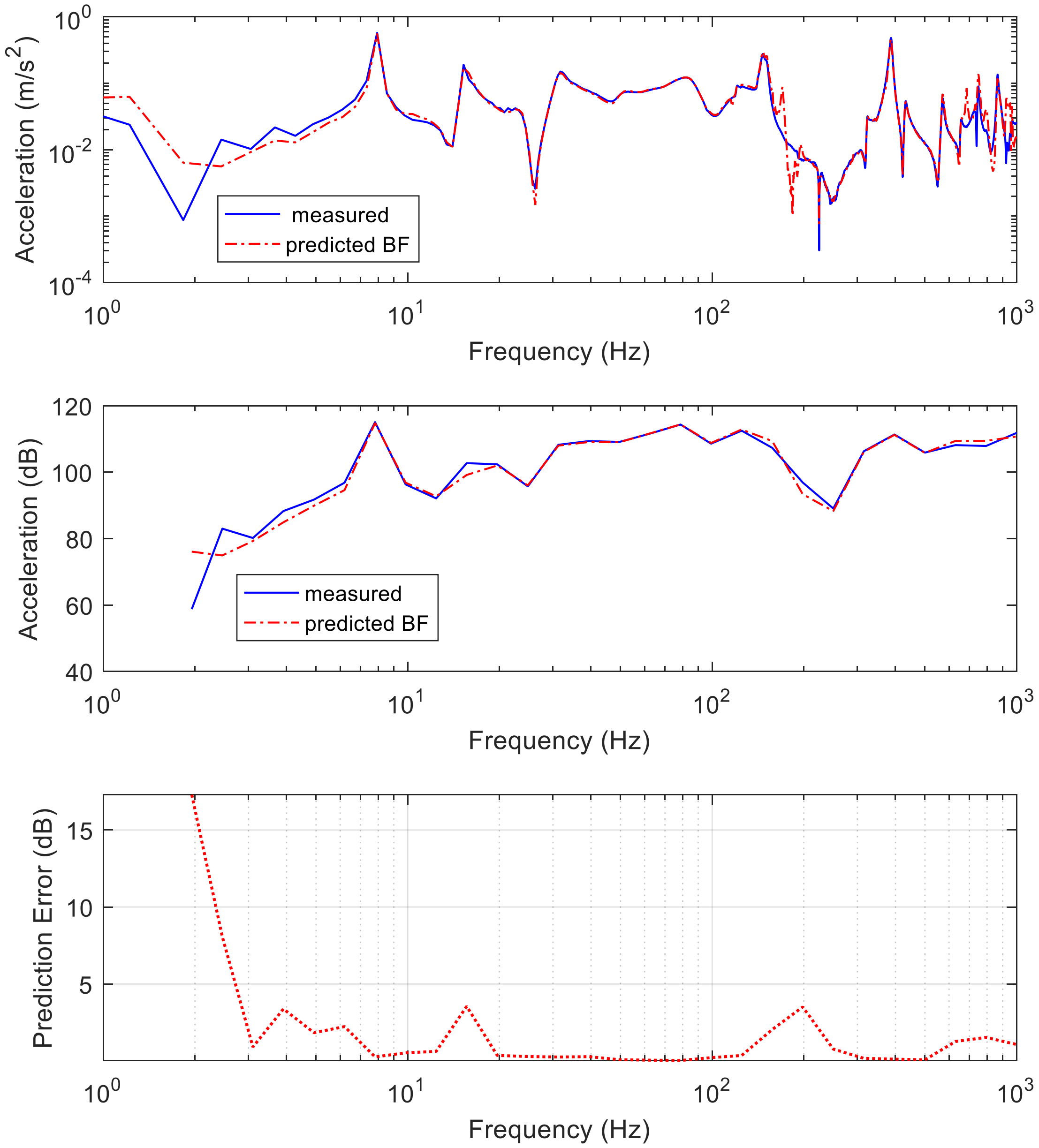


Figure 3: $a_{C,d}$ acceleration obtained in the coupled measurement (blue) vs prediction using the blocked force method (red-dashed) in narrow band frequency (top) in $m/s^2$ and per third octave band (middle) in decibels (ref. $1\mu m/s^2$). Bottom: error between prediction and *in situ* obtained coupled acceleration (dB) in third octave bands.

Results show that predictions of the source response using the blocked force method provide a very accurate fit of the *in situ* measured data in terms of both narrow band and third octaves. Errors between these measures are below 3.5dB above 3 Hz. The error observed at 1Hz (16dB) is due to a lack of coherence in the FRFs obtained below 10Hz.

These results validate the use of the blocked forces method as a reliable independent active source characterisation which is also descriptive of its performance in new assemblies.

### 3.2 Isolator characterisation

The material chosen as resilient coupling between source and receiver was VIDAM (detailed datasheet can be found in [12]), a nitrile rubber and granulated cork composite material typically used as industrial equipment vibration isolator. 40mm long and 30mm diameter cylindrical isolators of this material were characterised using two different methods: dynamic compression (Figure4-left) and *in situ* method (Figure4-right).

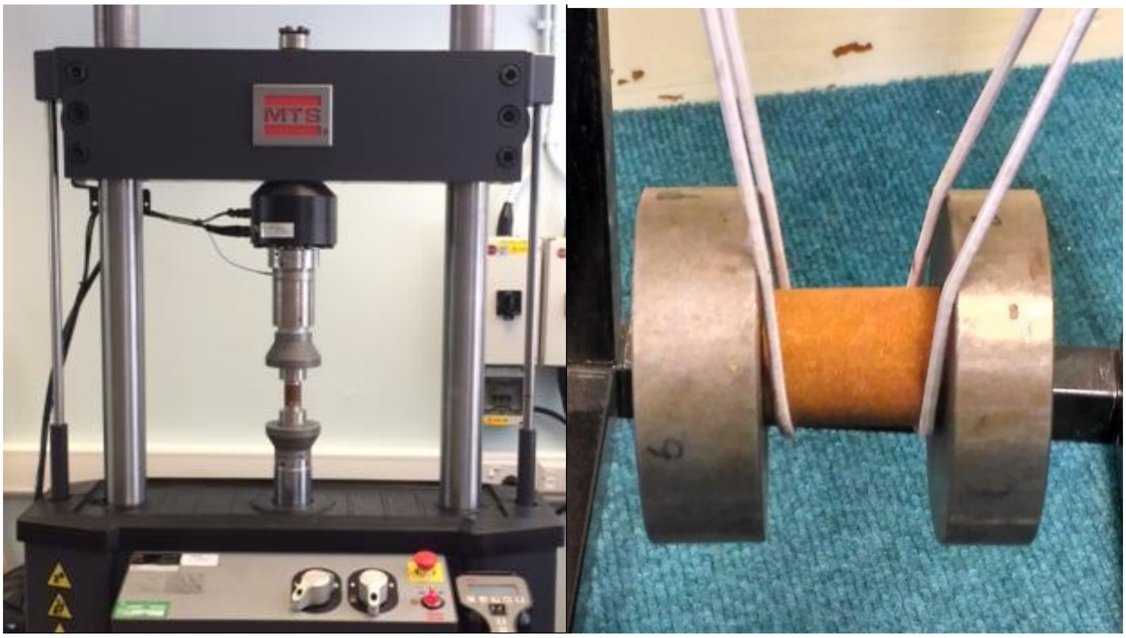


Figure 4: dynamic compression (left) and *in situ* (right) isolator characterisation experimental setups

The isolator was firstly characterised under dynamic compression at frequencies from 1 to 100 Hz using a dynamic hydraulic testing press (MTS, USA). The experimental conditions included an average static preload of 235.4N and a dynamic amplitude of 0.01mm. Preconditioning was applied to the sample prior to the experiment in order to obtain more accurate and repeatable results. As an outcome of this process, complex dynamic stiffness was obtained in eleven frequencies up to 100Hz.

In addition to the dynamic compression experiment, the isolator was also characterised using an *in situ* mass-isolator-mass procedure as described in [13]. It consisted of the acquisition of point and transfer mobilities above and below the material when connected to two known masses. Two single axis accelerometers were used for this experiment together with the application of forces above and below the two masses.

Dynamic stiffness was extracted from the accelerance FRFs acquired with the *in situ* method and then compared with the stiffness obtained using the dynamic hydraulic testing press in Figure 5.

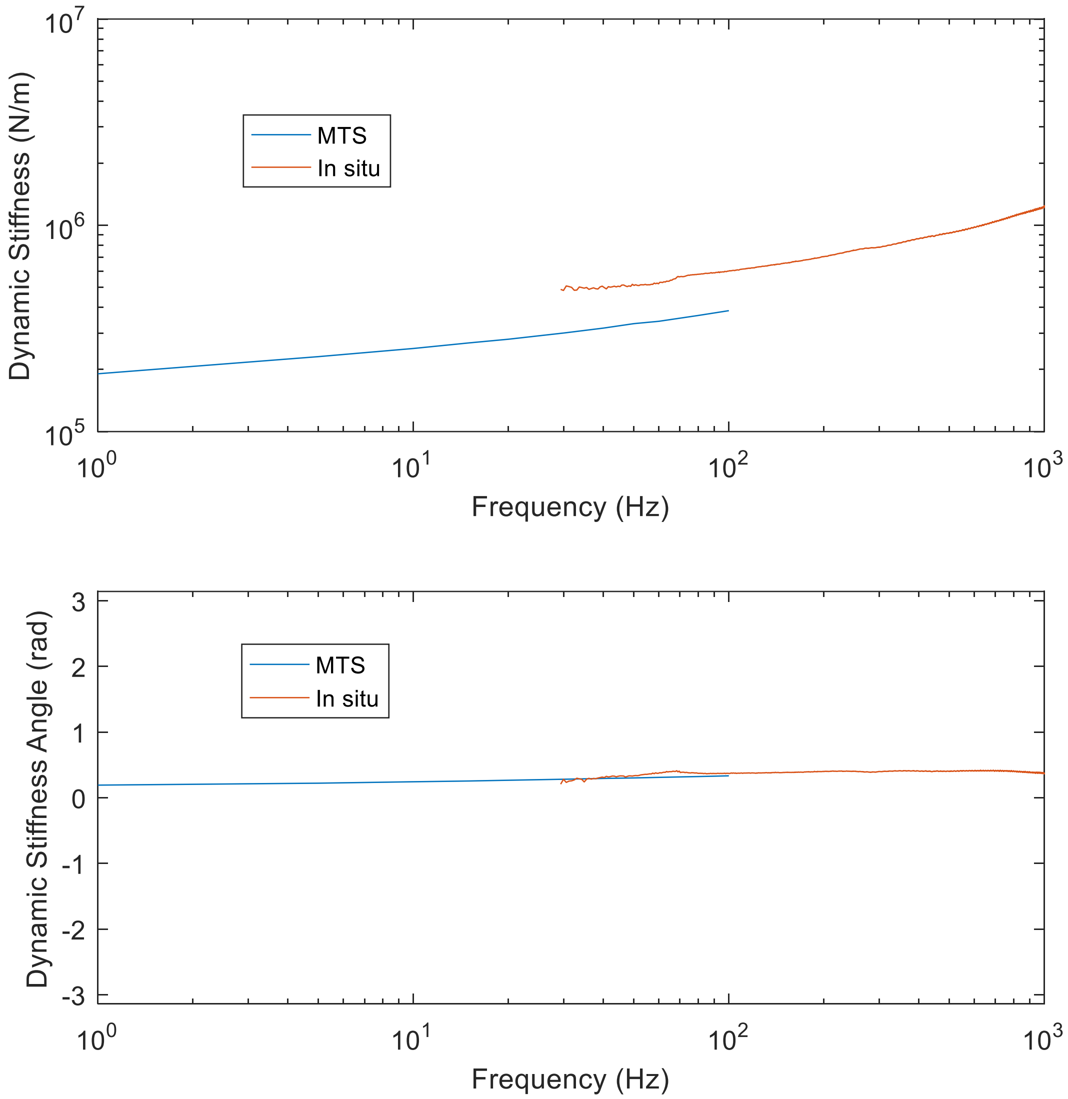


Figure 5: Dynamic stiffness of VIDAM isolator measured using the *in situ* Mass-isolator-mass procedure (red) vs. using Dynamic compression (blue) in both terms of magnitude (top) and phase (bottom).

Results show a similar trend followed by the two methods with some disagreement in terms of magnitude (Figure 5-top), which could be justified by the preload difference in the two experiments. Due to the lack of coherence in the accelerance FRFs obtained for the *in situ* method at the lowest frequencies (particularly below 10Hz), the isolator was characterised using dynamic compression results below 100Hz and the in situ experiment results for frequencies above.

## 3.3 Coupled Assembly

Once performed the assembly elements independent characterisations, a fourth experiment defined as the coupled assembly characterisation was performed by coupling the source and receiver via three VIDAM isolators glued at the same time to the three steel feet of the source and the receiver slab in the positions shown in Figure 6.

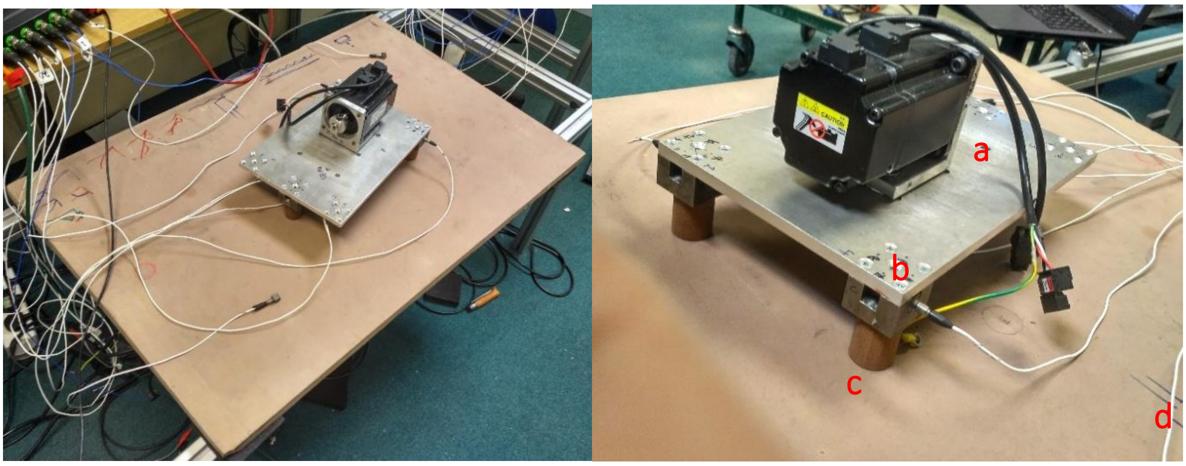


Figure 6: Left: Coupled Source-Isolator-Receiver assembly experimental setup. Right: Closer view of the assembly identifying the different interfaces (a-d) in red font.

A total of eleven accelerometers were used for the coupled experiment providing acceleration responses at both the source and receiver in identical positions to the independent experiments described previously. Excitations were applied to both the source (three on top of the isolator coupling points -b- and two internal to the source -a-) and the receiver structures (three below the isolator coupling points -c- and two external to the isolator-receiver interface -d-), obtaining this way a symmetric accelerance matrix as detailed in Eq. (4).

# 4 Results

As described in the theory section, dynamic sub-structuring is applied to the independently characterised source, isolator and receiver accelerance FRFs by mathematically coupling as described in Eq. (3). As an outcome of this, the predicted coupled accelerance at the external point in the receiver (interface d) is obtained after applying Eq. (4) to the resulting matrix and the two Boolean coupling matrices that assure conditions of equilibrium and compatibility.

## 4.1 Dynamic Sub-structuring predictions

Figure 7 graphically shows the comparison between the predicted coupled assembly accelerance and the *in situ* measured response of the coupled elements, in both terms of magnitude (Figure 7-top) and phase (Figure 7-bottom).

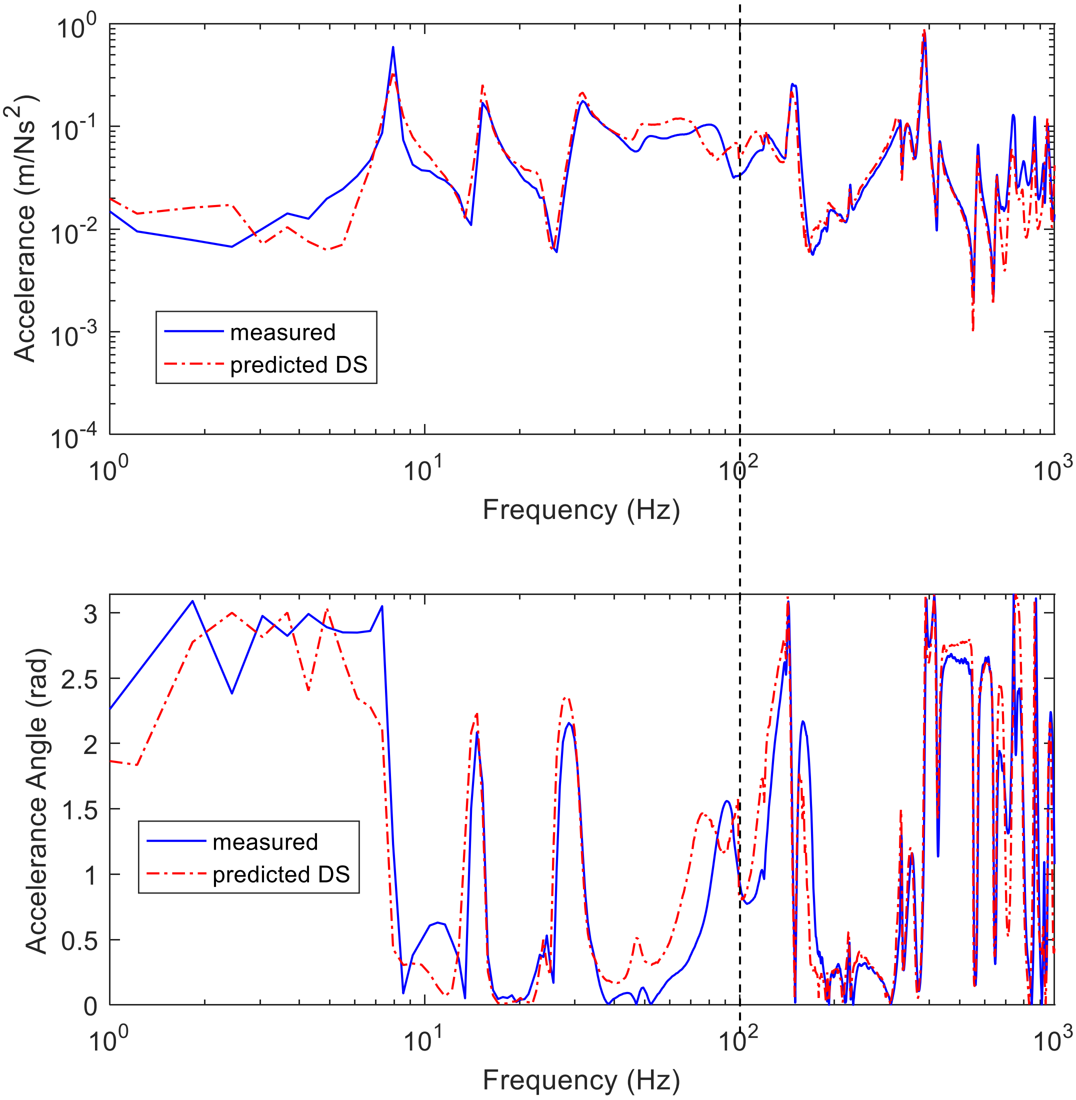


Figure 7: $A_{C,d}$ accelerance obtained in the coupled measurement (blue) vs prediction using the dynamic sub-structuring method (red-dashed) in terms of magnitude (top) and phase(bottom). Black dashed line divides the isolator characterisation between MTS (below 100Hz) and *in situ* methods.

Results show a very good agreement throughout the entire frequency range studied in terms of magnitude and phase, which includes isolator characterisation from two different methods detailed in the previous section (delimited by the black dashed line in Figure 7). Small disagreements were observed between 40 and 100Hz and below 10Hz, the latter being due to poor coherence in the FRFs obtained.

## 4.2 Blocked Forces and Dynamic Sub-structuring combined predictions

Once validated the individual application of blocked forces method and dynamic sub-structuring for the data collected in the different experiments, a combination of the two methods was implemented as described in Eq. (5), defining "$A_{C,db}$" as the DS prediction using the independent assembly element characterisations; and "$f_{Sb}$" as the blocked force extracted from the active source characterisation as detailed in section 3.1.

Figure 8 graphically compares the predicted acceleration at the interface d when exciting an internal part of the source (interface a) with the same acceleration measured *in situ* extracted from the coupled experiment.

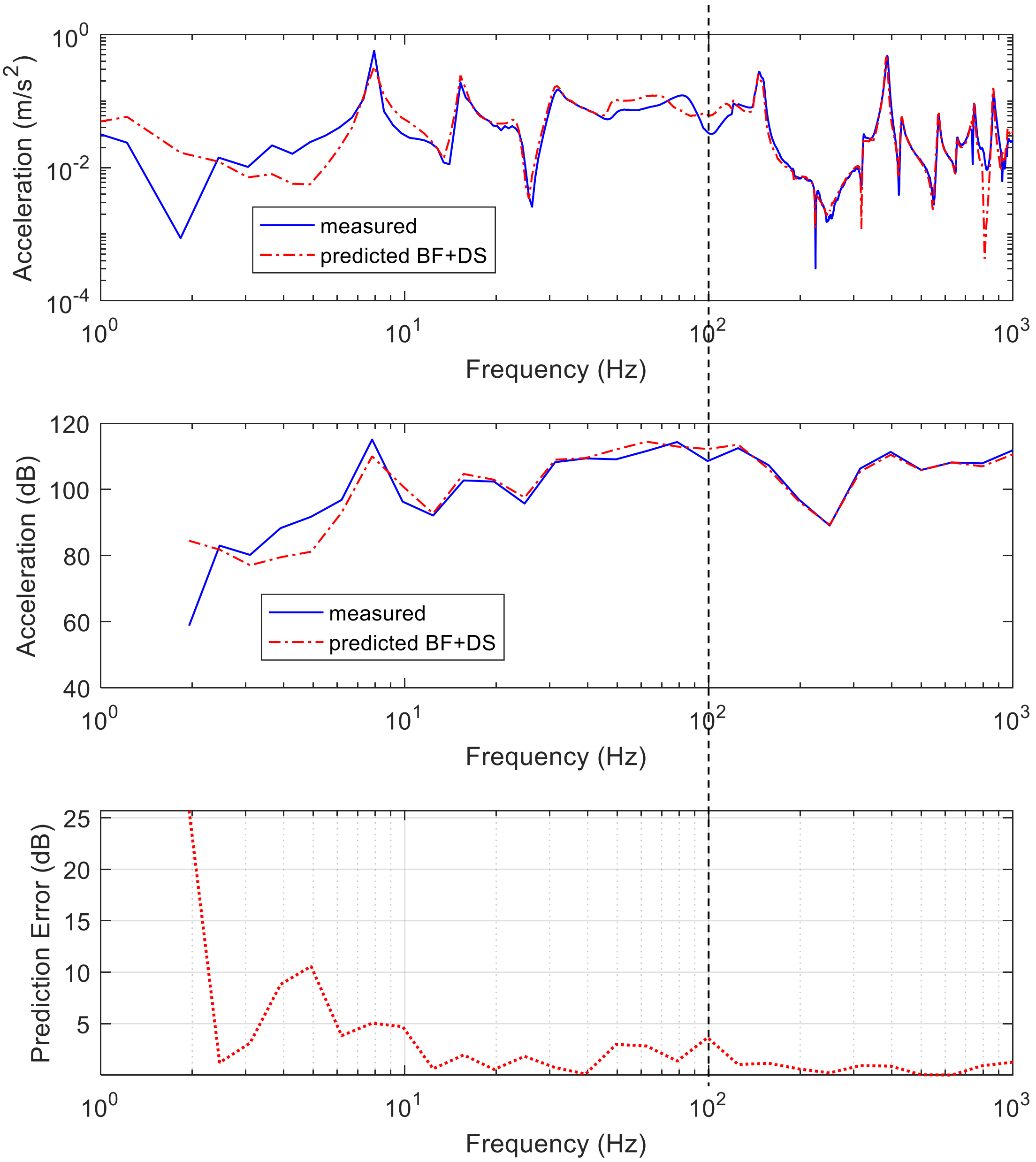


Figure 8: $a_{C,d}$ accelerance obtained in the coupled measurement (blue) vs prediction using the blocked forces method combined with dynamic sub-structuring (red-dashed) in terms of narrow band frequency (top) in m/s$^2$ and per third octave band (middle) in decibels (ref. 1µm/s$^2$). Bottom: error between prediction and *in situ* obtained coupled acceleration (dB) in third octave bands. Black dashed line divides the isolator characterisation between MTS (below 100Hz) and *in situ* methods.

The resulting curves exhibit a good agreement in both terms of narrow band frequencies and octave bands, exhibiting noticeable error peaks of 25.7dB and 10.6dB at 1.9Hz and 4.9Hz respectively. Errors were always lower than 5dB above the latter frequency.

# 5 Conclusions

The work presented here demonstrates that the behaviour of a coupled assembly can be predicted using data from independent experiments performed on its elements. In order to achieve this, the source was characterised by its blocked force and, together with the mobility matrices obtained for the resilient isolators and receiver, the coupled assembly FRF is constructed by applying the sub-structuring method.

Besides showing very good overall agreements, the methodology proposed provided some noticeable errors in the lowest frequencies studied. Measures that could compensate for these errors include the use of accelerometers with better low frequency response and the use of a bigger instrumentation hammer for better excitation of the low frequency structural modes in order to increase the coherence of the different FRFs acquired.

Future work includes the validation of this methodology to heavier structures more representative of realistic situations such as industrial equipment and concrete floors as well as its posterior application to real building vibration isolation. Alternatively, the use this methodology also allows the use of non-empirical data for the definition of independent measures of the assembly elements (such as FEA and/or SEA models) and therefore its implementation should be studied in more detail.

Furthermore, the application of structure decoupling could be used to extract independent assembly element properties by using coupled experiment data together with independent characterisation of the remaining assembly elements (either *in situ* or by the use of numerical methods).

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